Research on Autocorrelation and RFM Autocorrelation of Chaotic Sequence based on Phase Space Method

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***Abstract*—Chaotic sequences have been widely used as pseudorandom sequences. But up to now, we have no simple and effective method to test and improve their autocorrelation and RFM autocorrelation performances. In the paper, by method of phase space, relations between autocorrelation and phase space trajectory were studied, simple and effective methods were presented to assess and improve their autocorrelation and RFM autocorrelation performances. Many simulations were presented, too.**

***Keywords*—Chaos, Autocorrelation, Random Frequency Modulation (RFM), Phase space**

I. INTRODUCTION

Chaotic sequence has been widely used in communication and radar as pseudorandom sequence[1-25]. But we have no simple method to test and improve their autocorrelation and modulated autocorrelation performances. Therefore, applications of chaotic sequences are limited[1-18]. (For easy to describe, in the paper, autocorrelation of modulated signal is referred to as modulated autocorrelation.)

Recently, chaotic sequences were used in noise radar as noise source[1-11,18]. But because of structure of chaotic attractor, some chaotic sequences have poor autocorrelation performance, or after modulation, such as Random Frequency Modulation (RFM)[3-6], autocorrelation performance of modulated signal is poor[3-6]. This problem impede the detection and identification of chaos-based signals, so autocorrelation and modulated autocorrelation of chaotic sequence have attracted many academic attention [2-7].

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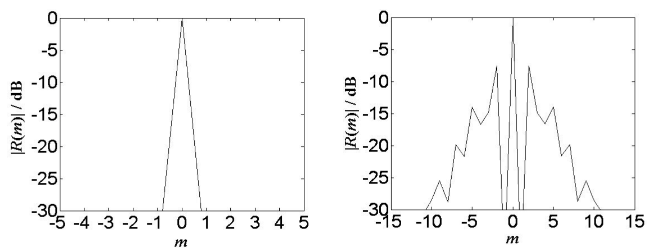
In the paper, by phase space, we proved some theorems to present simple and effective methods for assessment and improvement of autocorrelation and RFM autocorrelation performances of chaotic sequences. Simulations were presented to verify them. Too.

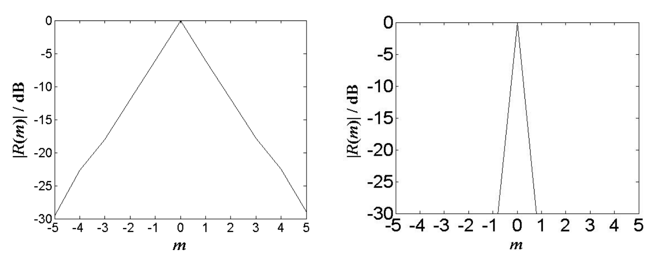
II. THE PROBLEM OF CHAOTIC SEQUENCE AUTOCORRELATION



(a) Normalized autocorrelation of Tent sequence

(b)Normalized autocorrelation of RFM modulated Tent sequence

Fig. 1. Normalized autocorrelation and RFM modulated autocorrelation functions of Tent sequence



(a) Normalized autocorrelation of Bernoulli sequence

(b)Normalized autocorrelation of RFM modulated Bernoulli sequence

Fig. 2. Normalized autocorrelation and RFM modulated autocorrelation functions of Bernoulli sequence

For communication, the shape of a good autocorrelation function ought to like a sharp needle, as shown in figure 1(a). It

has a sharp and outstanding main peak, and has no outstanding side lobe, so this signal can be easily detected and identified.

The autocorrelation function of Tent sequence is presented in figure 1(a), its main peak is sharp and outstanding, and its side lobe is low and can hardly be find out, so its performance is very good. Although Tent sequence autocorrelation is good, but after Random Frequency Modulation (RFM)[3-6], its RFM autocorrelation performance becomes bad, as shown in figure 1(b), there are many outstanding side lobes.

The autocorrelation function of Bernoulli sequence is shown in figure 2(a), its main peak is blunt and fat, so its performance is poor. But after RFM, its RFM autocorrelation performance becomes good, as shown in figure 2(b).

Therefore, some chaotic sequences have bad autocorrelation performance, and some chaotic sequences have poor modulated autocorrelation performance. So it needs to study this phenomenon, and find out its rule to assess and improve their autocorrelation and RFM autocorrelation performances for better applications.

III. AUTOCORRELATION OF CHAOTIC SEQUENCE

By Autocorrelation Phase-space Axis Symmetric(APAS) theorem[25], we can estimate autocorrelation performance of a sequence. It only needs to test whether its trajectory in delay 1 phase space is x-axis or y-axis symmetrical. If its trajectory is x-axis or y-axis symmetrical, then its autocorrelation performance is good, namely, its autocorrelation has sharp and outstanding main peak, and has no remarkable sidelobe.

we list APAS theorem as follows.

Theorem 1: A stationary ergodic discrete real dynamic system

$$x(n+1) = \varphi[x(n)] \tag{1}$$

Where $\varphi(x)$ is a single valued function, value area of $x(n)$ is [-a, a], and 'a' is a positive real number; the mean of $\{x(n)\}$ is zero. values of $x(n)$ are statistically balanced, namely $f[x(n)] = f[-x(n)]$, where, $f[x(n)]$ is probability density function of x(n).

If $\varphi(x)$ satisfy any one of following condition

$$x(n+1) = \varphi[x(n)] = \varphi[-x(n)] \tag{2}$$

$$f[x(n), x(n+1)] = f[-x(n), x(n+1)] \tag{3}$$

Then, we get following conclusion

when $N \to \infty$, and delay $m \neq 0$, to any other delay m, the normalized autocorrelation function of $\{x(n)\}$ is near to zero. That is

$$\lim_{N\to\infty} R(m) = \lim_{N\to\infty} \frac{1}{r(0)} \sum_{n=1}^{N-1} x(n)x(n+m) = 0 \qquad m \in N,\ \ m \neq 0 \tag{4}$$

Theorem 2: A stationary ergodic discrete real dynamic system

$$x(n+1) = \varphi[x(n)] \tag{5}$$

Where inverse function $\varphi^{-1}(x)$ of $\varphi(x)$ is single valued. Value area of $x(n)$ is [-a, a], and 'a' is positive real; the mean of $\{x(n)\}$ is zero. values of $x(n)$ are statistically balanced, namely $f[x(n)] = f[-x(n)]$, where, $f[x(n)]$ is probability density function of $x(n)$.

If $\varphi(x)$ satisfy any one of following condition

$$x(n) = \varphi^{-1}[x(n+1)] = \varphi^{-1}[-x(n+1)] \tag{6}$$

$$f[x(n), x(n+1)] = f[x(n), -x(n+1)] \tag{7}$$

Then, we get following conclusion

when $N \to \infty$, and delay $m \neq 0$, to any other delay m, the normalized autocorrelation function of $\{x(n)\}$ is near to zero. That is

$$\lim_{N\to\infty} R(m) = \lim_{N\to\infty} \frac{1}{r(0)} \sum_{n=1}^{N-1} x(n)x(n+m) = 0 \qquad m \in N,\ \ m \neq 0 \tag{8}$$

Combining Theorem 1 and Theorem 2, we call them as Autocorrelation Phase-space Axis Symmetric(APAS) Theorem.

APAS theorem was proved and verified in paper[25].

From APAS theorem we can see, to estimate a sequence autocorrelation performance, it only needs to test whether its delay 1 phase space trajectory is x-axis or y-axis symmetrical. If its trajectory is x-axis or y-axis symmetrical, then its autocorrelation performance is good, and has sharp and outstanding main peak, and has no remarkable sidelobe.

We have assessed and improved autocorrelation performances of some chaotic sequences[26,27] by APAS theorem, it is verified that APAS theorem is simple and effective.

IV. RFM AUTOCORRELATION OF CHAOTIC SEQUENCE

Recently, Random Frequency Modulation(RFM) received more and more academic attention [3-7].

A discrete chaotic system, its dynamic equation is

$$x(n+1) = \varphi[x(n)] \tag{9}$$

Value area of $x(n)$ is [-a, a], and 'a' is maximum of x(n), is a positive real number; the mean of $\{x(n)\}$ is zero. values of $x(n)$ are statistically balanced, namely $f[x(n)] = f[-x(n)]$, where, $f[x(n)]$ is probability density function of x(n).

Discrete RFM signal of {x(n)} with complex envelope[3,4]

$$s(i) = A\exp[\ j2\pi K\sum_{n=0}^{i} x(n) + \ j2\pi\varphi_0] \qquad (10)$$

Where K is an index which combine modulation index and sample rate, is a positive real number.

For satisfy the Nyquist criterion, must meet condition of

$$2Ka \le 1 \qquad (11)$$

We choose

$$2Ka = 1 \qquad (12)$$

Then, phase angle of $\exp[j2\pi Kx(n)]$ is distributed in a single full cycle [-π, π]. In order to convenient for description, we call equation (12) as Single Full Cycle(SFC) condition. Without loss of generality, we choose k=1, a=0.5, and following simulations all satisfy the condition.

A stationary ergodic sequence {x(n)} with length N, its RFM autocorrelation function r(m) is

$$r(m) = \sum_{n=1}^{N-m} s(n)s^*(n+m) \qquad (13)$$

Normalized RFM autocorrelation function R(m) is

$$R(m) = \frac{1}{r(0)}\sum_{n=1}^{N-m} s(n)s^*(n+m) \qquad (14)$$

RFM-APTS Theorem: A stationary ergodic discrete real dynamic system

$$x(n+1) = \varphi[x(n)] \qquad (15)$$

Value area of $x(n)$ is [-a, a], the mean of {$x(n)$} is zero. values of $x(n)$ are statistically balanced, namely $f[x(n)] = f[-x(n)]$, where, $f[x(n)]$ is probability density function of x(n). RFM signal of {x(n)} satisfy SFC condition.

On delay 1 trajectory of {$x(n)$}, if the trajectory is statistically translational symmetrical, as shown in figure 3, namely

$$\begin{cases} x(n+1) = \varphi[x(n)] = \varphi[x(n)+a] \\ \qquad\qquad\qquad\qquad x(n) < 0 \\ x(n+1) = \varphi[x(n)] = \varphi[x(n)-a] \\ \qquad\qquad\qquad\qquad x(n) \ge 0 \end{cases} \qquad (16)$$

and

$$\begin{cases} f[x(n)] = f[x(n)+a] \\ \qquad\qquad\qquad\qquad x(n) < 0 \\ f[x(n)] = f[x(n)-a] \\ \qquad\qquad\qquad\qquad x(n) \ge 0 \end{cases} \qquad (17)$$

Where, f[x(n)] is probability density function of x(n)。

Then, we get following conclusion

when $N \to \infty$, and delay $m \ne 0$, to any other delay m, the normalized RFM autocorrelation function of {$x(n)$} is near to zero. That is

$$\lim_{N\to\infty} R(m) = \lim_{N\to\infty} \frac{1}{r(0)}\sum_{n=1}^{N-1} s(n)s^*(n+m) \approx 0 \qquad (18)$$

$$m \in N,\quad m \ne 0$$

Proof:

(1) prove $\lim_{N\to\infty} R(1) = 0$.

On delay 1 phase trajectory, we can arbitrary choose a couple of points $D(k)$、$D(j)$ which are translational symmetrical. $D(k)$ is on right part of phase plane, and coordinate is [$x_1(k)$, $x_1(k+1)$]; $D(j)$ is on left part of phase plane, and coordinate is [$x_1(j)$, $x_1(j+1)$]. Then, have $x_1(j)+a = x_1(k)$, $x_1(k+1) = x_1(j+1)$, as shown in figure 3. Figure 3 is an example figure using Bernoulli sequence as an illustration.

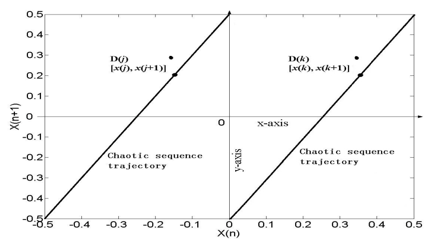


Fig. 3. delay 1 phase trajectory of Bernoulli sequence

Because
$$\begin{cases} f[x(n)] = f[x(n)+a] \\ \qquad\qquad\qquad\qquad x(n) < 0 \\ f[x(n)] = f[x(n)-a] \\ \qquad\qquad\qquad\qquad x(n) \ge 0 \end{cases}$$

Then, without loss of generality, choose $x(n) \ge 0$,

so $f[x(n)] = f[x(n)-a]$。

Let $P[x(n+1) = x_1(k+1) \mid x(n) = x_1(k)]$, $f[x(n+1) = x_1(k+1) \mid x(n) = x_1(k)]$ are conditional probability and conditional probability density function of $x(n+1) = x_1(k+1)$ under condition $x(n) = x_1(k)$ respectively. In the paper, they are abbreviated as $P[x_1(k+1) \mid x_1(k)]$, $f[x_1(k+1) \mid x_1(k)]$.

To a point $D(k)[x_1(k), x_1(k+1)]$ on trajectory, because $x_1(k+1) = \varphi[x_1(k)]$, yields

$$P[x_1(k+1) \mid x_1(k)] = P[x_1(k+1) \mid x_1(k) - a] = 1$$

Then

$$f[x_1(k+1) \mid x_1(k)] = f[x_1(k+1) \mid x_1(k) - a]$$

Let $f[x(n) = x_1(k), \quad x(n+1) = x_1(k+1)]$ is Joint probability distribution density function of $x(n) = x_1(k)$ and $x(n+1) = x_1(k+1)$, and is abbreviated as $f[x_1(k), \quad x_1(k+1)]$.

So, on trajectory, to translational symmetrical couple of points $D(k)$、$D(j)$, have

$$\begin{aligned} & f[x_1(k), \quad x_1(k+1)] \\ & = f[x_1(k+1) \mid x_1(k)] \times f[x_1(k)] \\ & = f[x_1(k+1) \mid x_1(k) - a] \times f[x_1(k) - a] \\ & = f[x_1(j), \quad x_1(j+1)] \end{aligned} \tag{19}$$

That is, distribution probability densities of the two points are identical, namely $f[D(k)]=f[D(j)]$.

To a point $D(k)$ $[x_1(k), x_1(k+1)]$,

Let function $h(k)$ is

$$\begin{aligned} h(k) &= s(k-1)s^*(k) \\ &= A^2 \exp[-j2\pi Kx(k)] \end{aligned} \tag{20}$$

The couple of points $D(k)$、$D(j)$, satisfy SFC condition, we get

$$\begin{aligned} h(j) &= A^2 \exp[-j2\pi Kx(j)] \\ &= A^2 \exp[-j(2\pi Kx(k) + 2\pi Ka)] \\ &= -A^2 \exp[-j2\pi Kx(k)] \\ &= -h(k) \end{aligned} \tag{21}$$

Then, we get $h(k) + h(j) = 0$.

On trajectory, to any near translational symmetrical two points $D(k)$、$D(j)$, have $h(k) + h(j) \approx 0$.

Recall that translational symmetrical two points $D(k)$、$D(j)$ satisfy $f[D(k)]=f[D(j)]$

Then, all the (N-1) points on the phase figure can be approximately divided into (N-1)/2 groups, and two points of each group are near translational symmetrical, and to each group $D(k_n)$、$D(j_n)$, satisfy $h(k_n) + h(j_n) \approx 0$.

There are few points can't be grouped as they are not translational symmetrical or number of points is odd number. But distribution probability densities of any two points on trajectory are identical, when $N \to \infty$, the distribution of points on trajectory tends to be more close to translational symmetrical, whereas number of points which can't be grouped is limited, and because of

$$\lim_{N\to\infty} R(1) = \lim_{N\to\infty} \frac{1}{r(0)} \sum_{n=1}^{N-1} s(n)s^*(n+1)$$

$$= \frac{A^2}{r(0)} \sum_{n=1}^{N-1} \exp[-j2\pi Kx(n+1)] = \lim_{N\to\infty} \frac{1}{r(0)} \sum_{n=1}^{N-1} h(n+1)$$

Denominator $\lim_{N\to\infty} r(0) = \lim_{N\to\infty} \sum_{n=1}^{N-1} |s(n)|^2 \to +\infty$

So, the effects of no grouped points to $R(1)$ tend to be zero,

We get

$$\begin{aligned} \lim_{N\to\infty} R(1) &= \lim_{N\to\infty} \frac{1}{r(0)} \sum_{n=1}^{N-1} h(n+1) \\ &\approx \frac{1}{+\infty} \lim_{N\to\infty} \sum_{n=1}^{(N-1)/2} 0 \approx 0 \end{aligned}$$

(2) prove $\lim_{N\to\infty} R(m) \approx 0$

Based on $\lim_{N\to\infty} R(1) \approx 0$, Similar to the proof of APAS Theorem[25], we can obtain $\lim_{N\to\infty} R(m) \approx 0$.

Which completes the proof.

V. Conclusions about Assessment of Autocorrelation and RFM Autocorrelation

Sum up, we get conclusions that,

(1)Assessment of autocorrelation:

A stationary ergodic statistically balanced time sequence which mean is zero, if its delay 1 phase trajectory is statistically axial symmetrical,

Then, its autocorrelation performance are good.

(2)Assessment of RFM autocorrelation:

A stationary ergodic statistically balanced time sequence

which mean is zero and satisfy SFC condition, if its delay 1 phase trajectory is statistically translational symmetrical,

Then, its RFM performance are good.

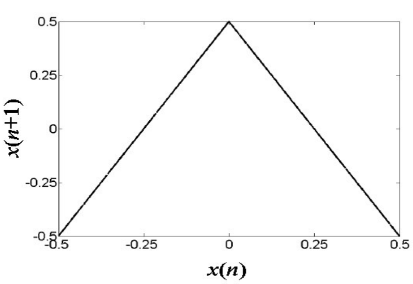
From APAS and RFM-APTS theorems we can see, to estimate a sequence autocorrelation and RFM autocorrelation performances, only needs to test the shape of delay 1 phase space trajectory, if it is x-axis or y-axis symmetrical, then autocorrelation performance is good; if it is translational symmetrical, then RFM autocorrelation performance is good.

Phase space trajectory figure is an effective tool to observe a dynamic system internal characteristic, by which we can recognize structure of the system. Generally, compared with other larger delay phase space figure, the structure of delay 1 phase space figure is far more simple and clear, so APAS and RFM-APTS theorems is a simple and effective method to judge a sequence autocorrelation and RFM autocorrelation performances.

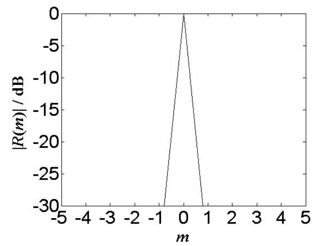
VI. Testing Autocorrelation and RFM Autocorrelation Performances by APAS and RFM-APTS Theorems

Tent sequence phase trajectory is shown in figure 4(a), Its trajectory is y-axis symmetrical, satisfies axial symmetrical condition, but doesn't satisfy translational symmetrical condition. By APAS and RFM-APTS theorems, its autocorrelation performance is good, RFM autocorrelation performance is poor. Its normalized autocorrelation and RFM autocorrelations are shown in figure 4(b), (c), and meet APAS and RFM-APTS theorems.

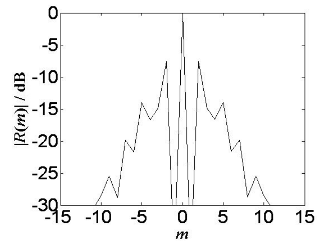
Delay 1 phase trajectory of Bernoulli sequence is shown in figure 5(a), its trajectory doesn't satisfy axial symmetrical condition, but satisfies translational symmetrical condition. By APAS and RFM-APTS theorems, its autocorrelation performance is poor, RFM autocorrelation performance is good. Its normalized autocorrelation and RFM autocorrelations are shown in figure 5(b), (c), and meet APAS and RFM-APTS theorems.



(a) Delay 1 phase space trajectory of Tent sequence

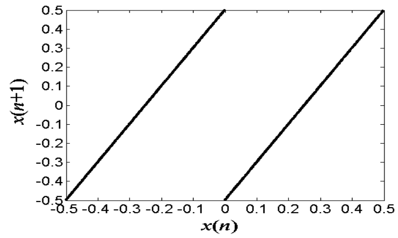


(a) Normalized autocorrelation of Tent sequence

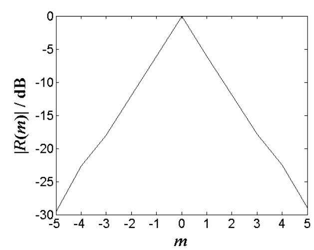


(b)Normalized RFM autocorrelation of Tent sequence

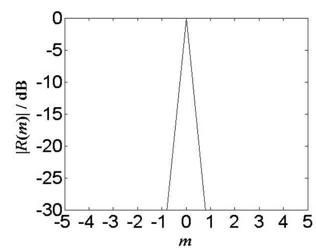
Fig.4. Delay 1 phase space trajectory of Tent sequence and its autocorrelation and RAM, RFM, RPM autocorrelation functions



(a) Delay 1 phase space trajectory of Bernoulli sequence



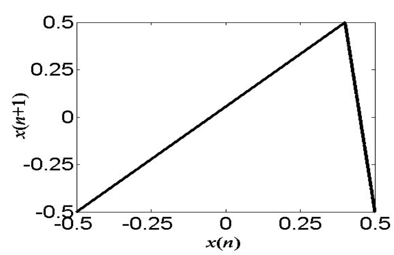
(b) Normalized autocorrelation of Bernoulli sequence



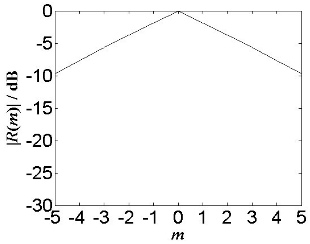
(c)Normalized RFM autocorrelation of Bernoulli sequence

Fig. 5. Delay 1 phase space trajectory of Bernoulli sequence and its autocorrelation and RAM, RFM, RPM autocorrelation functions

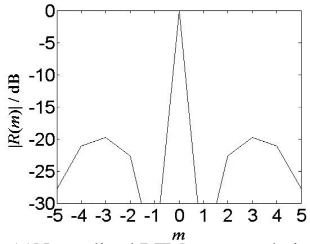
Skew Tent sequence is different from upper Tent and Bernoulli sequences. Delay 1 phase trajectory of Skew Tent sequence is shown in figure 6(a), its trajectory neither satisfies axial symmetrical condition, nor satisfies translational symmetrical condition. By APAS and RFM-APTS theorems, its autocorrelation and RFM autocorrelation performances all are poor. Its normalized autocorrelation and RFM autocorrelations are shown in figure 6(b), (c), and meet APAS and RFM-APTS theorems.



(a) Delay 1 phase space trajectory of Skew Tent map which vertex is 0.4



(b) Normalized autocorrelation of Skew Tent sequence



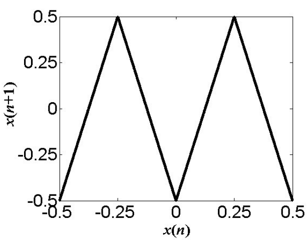
(c)Normalized RFM autocorrelation of Skew Tent sequence

Fig. 6. Delay 1 phase space trajectory of Skew Tent sequence and its autocorrelation and RAM, RFM, RPM autocorrelation functions

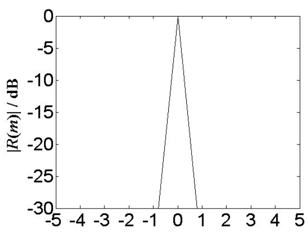
VII. Improvement of Autocorrelation and RFM Autocorrelation Performances by APAS and RFM-APTS Theorems

To a chaotic map which trajectory is not axial or translational symmetrical, that doesn't satisfy conditions of APAS and RFM-APTS theorems, we can modify its map, by methods of compressing, copy, fold, mirror, translation, etc, change its attractor to satisfy axial and translational symmetrical conditions, then the modified chaotic sequences have good autocorrelation and RFM autocorrelation performances.

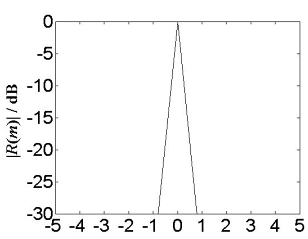
To tent map, its trajectory is axial symmetrical, so we modify its map, copy, compress and translate its trajectory, we call the modified map as Axial and Translational Symmetrical Tent(ATS-Tent) map. Phase trajectory of ATS-Tent is shown in figure 7(a), is Axial and Translational Symmetrical, therefore its autocorrelation and RFM autocorrelation performances are good, as shown in figure 7(b),(c), and meet APAS and RFM-APTS theorems.



(a) phase space trajectory of ATS-Tent sequence



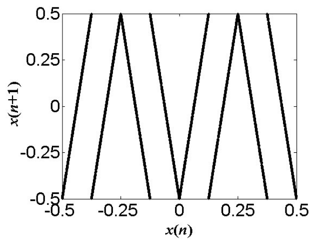
(b) Normalized autocorrelation of ATS-Tent sequence



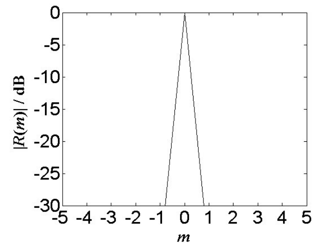
(c) Normalized RFM autocorrelations of ATS-Tent sequence

Fig.7. Delay 1 phase space trajectory of ATS-Tent sequence and its autocorrelation and RFM autocorrelation functions

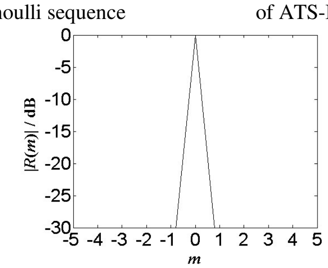
To Bernoulli map, we copy or mirror its trajectory two times, We call the map as Axial and Translational Symmetrical Bernoulli(ATS-Bernoulli) map, its delay 1 phase figure is shown in figure 8(a). It is Axial and Translational Symmetrical, then its autocorrelation and RFM autocorrelation performances all are good, as shown in figure 8(b), (c), and meet APAS and RFM-APTS theorems.



(a) phase space trajectory of ATS-Bernoulli sequence



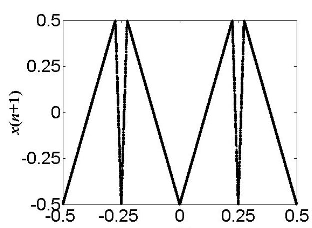
(b) Normalized autocorrelation of ATS-Bernoulli sequence



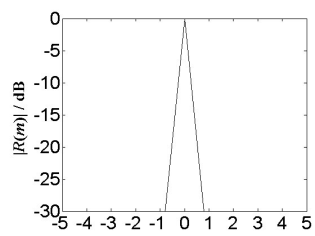
(c) Normalized RFM autocorrelation of ATS-Bernoulli sequence

Fig. 8. Delay 1 phase space trajectory of ATS-Bernoulli sequence and its autocorrelation and RFM autocorrelation functions

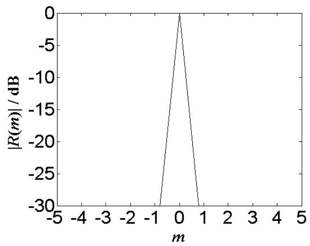
To Skew tent map, its trajectory neither is axial symmetrical, nor translational symmetrical. We copy or mirror its trajectory two times, We call the map as Axial and Translational Symmetrical Skew tent(ATS-Skew tent) map, its delay 1 phase figure is shown in figure 9(a). It is Axial and Translational Symmetrical, then its autocorrelation and RFM autocorrelation performances all are good, as shown in figure 9(b),(c), and meet APAS and RFM-APTS theorems.



(a) phase space trajectory of ATS- Skew Tent sequence



(b) Normalized autocorrelation of ATS- Skew Tent sequence



(c) Normalized RFM autocorrelation of ATS- Skew Tent sequence

Fig. 9. Delay 1 phase space trajectory of ATS- Skew Tent sequence and its autocorrelation and RFM autocorrelation functions

VIII. CONCLUSION

By phase space method, we presented APAS and RFM-APTS theorems for assessment and improvement of autocorrelation and RFM autocorrelation performances of chaotic sequences. It is a simple and effective method by APAS and RFM-APTS theorems to assess and improve performances of autocorrelation and RFM autocorrelation. Simulations were presented to verify the theorem and the method, too.

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